# Easy Animations

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If a picture is worth 1,000 words, then a good animation is like a classic short story—a simple tale simply told. Animations are particularly effective in the teaching of mathematics because motion is often fundamental to the concept at hand, and a well-designed animation is usually an excellent way to introduce such a concept. In this chapter, we describe two ways to make animations and post them on your course websites. Once you master the process, you will be surprised at how easy it is to build and post your own animations. (Note: This article is available in two ways—as part of these conference proceedings and on the web. If you have access to the Internet, we recommend that you now switch to the online version of this article. It is available at http://math.bu.edu/people/paul/nsf_ccli_presentation_2004.html. The animations in the online version are live, and you will be able to work with them just as students do. The online version of this chapter also provides access to a number of files that help produce these animations.)

## Making a QuickTime® Movie

A QuickTime® movie is a wonderful way to encapsulate an animation and export it to the Web while giving the student some control over the illustration. Once the student has downloaded Apple's free QuickTime® player (available for PCs running Windows and Macs) and installed it for their browser, clicking on a movie yields a website that contains the animation along with a slider that can be used to control the action. One button starts the animation from the beginning, another single steps through the animation, and a third single steps in the opposite direction. The student can also use the slider to move in either direction at any speed.

As an example, consider the concept of arc length of a curve in space. We define this length as a limit of sums of lengths of line segments between points on the curve (Figure 1). As the number of points increases and they become more closely spaced, the sum of the lengths of the segments approaches a limiting value, and this value is the arc length of the curve.

In the animation shown in Figure 1, the slider represents the number of subdivisions of the curve. Moving the slider from left to right increases the number of subdivisions. At the same time, the sum of the lengths of the line segments is updated, and an observant student will note that this approximation is 32.64 (to two decimal places) as soon as there are at least 63 line segments that approximate the curve.

What was involved in constructing this animation? The frames of the animation were all rendered using Mathematica®. First, we defined a few graphics functions to handle some of the standard aspects of a three-dimensional figure, e.g., the axes, the tick marks, and the viewpoint for the figure. Then we plotted the curve. Finally, we added the

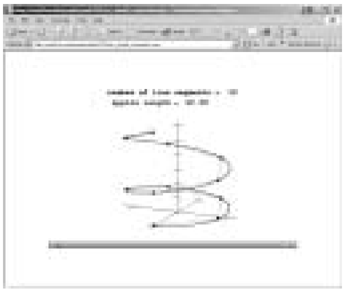

*Figure 1. QuickTime® arc length animation.*

points, their connecting line segments, and the labels that give the number of segments and the approximate arc length. A "Do" statement was used to produce one figure for each number of subdivisions between 10 and 100. The result was 91 images within Mathematica®. The images were animated within Mathematica® to get some sense of what the final animation would look like, and then they were exported as individual GIF files, one file for each image.

To produce the QuickTime® movie, we use QuickTime® Pro, which is an inexpensive upgrade to the free QuickTime® player. Launch QuickTime® Pro and select Open Image Sequence from the File menu. Go to the folder that contains the images and click on Image 1. Next, a frame rate is selected. We usually use 6 or 10 frames per second. Finally, one saves the movie by selecting Save As. We always click the box to make the movie self-contained, and, *voila,* the animation is saved to disk ready for posting to the appropriate course webpage.

Although we use Mathematica® to produce our animations, this technique works with any application (such as Maple or MATLAB) that can save images in any of a number of standard graphics formats (BMP, GIF, JPEG, PNG, TIFF, etc.).

In addition to the QuickTime® animations available from our websites, there are many wonderful ones available at other sites. In particular, we have been inspired by those that are available at Professor Louis A. Talman's website (1).

## LiveGraphics3D

For animations involving three-dimensional objects, there is another approach that gives the student even greater control. Martin Kraus (2), a postdoctoral fellow at Purdue University, wrote and maintains a Java 1.1 applet that gives the student the ability to manipulate three-dimensional graphics directly within any browser that supports Java 1.1, and this applet can be used without charge for noncommercial purposes. The graphic image is produced using Mathematica®, but the student does not need Mathematica ® or any special software to view and manipulate it.

For example, consider a circular hyperboloid of one sheet (Figure 2). Once rendered in Mathematica®, it only takes a few minutes to export the Mathematica® description of the surface into a file and post it on the web. Then the student can view the image, rotate it, resize it, and even spin it without any special kind of software.

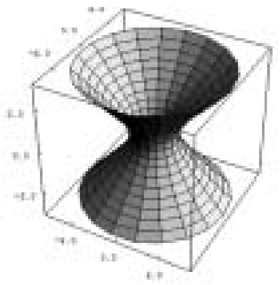

*Figure 2. Circular hyperboloid rendered using LiveGraphics3D.*

After you have used Mathematica® to produce a three-dimensional object that you want to post on the web, the rest of the process is straightforward. Within Mathematica®, you load the LiveGraphics3D package (available from Kraus's website [3]). This package provides the WriteLiveForm[] function. Applying this function to the graphic object produces an ASCII file that contains the Mathematica® description of the object. In the case of the circular hyperboloid shown above, this 100-K file resembles the following:

```
Graphics3D[{
Polygon[{{3.1623,  0.,  -3.},  {3.0883,
0.6798, -3.},
{2.8974,  0.63776,  -2.7931},  {2.9667,
0., -2.7931}}],
Polygon[{{3.0883, 0.6798, -3.}, {2.87,
1.3278, -3.},
{2.6925,  1.2457,  -2.7931},  {2.8974,
0.63776, -2.7931}}],
... (many polygons omitted) ...
Polygon[{{2.8974,  -0.63776,  2.7931},
{2.9667, -7.2661*^-16,2.7931},
{3.1623, -7.7451*^-16, 3.}, {3.0883, -
0.6798, 3.}}]]},
{PlotRange -> Automatic,
DisplayFunction :> $DisplayFunction,
... (many graphics options omitted) ...
BoxStyle -> Automatic,
SphericalRegion -> False}
]
```

Basically, it is a long list of polygons followed by a list of graphics options. In addition to the file that contains the Mathematica® description of the three-dimensional object, you need an HTML file that loads the Java applet, sets the values of a few parameters for the applet, and specifies the name of the file that contains the Mathematica® description. The structure of this file is specified on Kraus's website, but the quickest way to make this file is to copy the HTML source file for a LiveGraphics3D object that is already posted on the web and then change the character string for the file name that contains the Mathematica® description of the object. Note that this HTML file can contain additional HTML code.

Finally, the Java archive file, live.jar, with Kraus's applet must be available to the student on your website even though the student does not have to do anything special to download it. The browser loads it automatically when the student clicks on the HTML file, but you must make sure that the browser can find the applet.

## Animations with LiveGraphics3D

LiveGraphics3D can also render an animation that can be rotated and spun as well. Figure 3 shows an animation that includes a curve in the form of a trefoil knot along with a tangent vector that moves along the curve according to its parametrization. The student is able to control the animation just like a QuickTime® animation by moving the mouse left and right over the image while the right mouse button is pressed.

Making a LiveGraphics3D animation is much like making a QuickTime® animation. You produce the animation using Mathematica®, but in this case, you do not export the images. Instead, you use the WriteLiveAnimationForm[ ] command from the LiveGraphics3D package. This command produces a file that contains the Mathematica® description of the entire animation. Once again, you need an HTML file that loads Kraus's applet and specifies the name of the file with the Mathematica description®, but it has the same format as any other LiveGraphics3D HTML file.

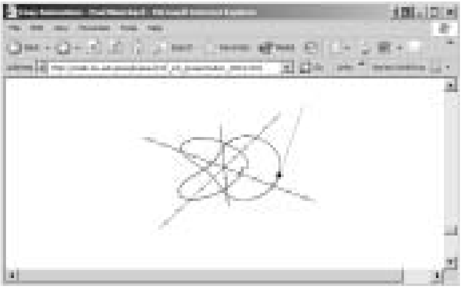

*Figure 3. Animation involving a trefoil knot rendered using LiveGraphics3D.*

## More Examples

We have produced a number of QuickTime® and LiveGraphics3D animations intended for the multivariable calculus and differential equation courses. There is a website (4) that links to many of them available on our website, and you can see how we use these animations by visiting the demo sections of the websites for our multivariable calculus (5) and differential equations (6) courses.

## The Hard Part

By now, it should be clear that the hard part of producing QuickTime® and LiveGraphics3D animations is designing the animation and producing it within Mathematica®. Once you have an animation there, making it available to your students either in QuickTime® format or using the LiveGraphics3D applet is easy.

### REFERENCES

1. http://clem.mscd.edu/~talmanl/MathAnim.html
2. http://www.vis.uni-stuttgart.de/~kraus/index.html
3. http://www.vis.uni-stuttgart.de/~kraus/LiveGraphics3D/mma_code/LiveGraphics3D.m
4. http://math.bu.edu/people/paul/225/nsf_ccli_presentation_2004_list.html
5. http://math.bu.edu/people/paul/MA225-fall01.html
6. http://math.bu.edu/people/paul/MA226-spring03.html