What I Wish I Had Known in Art School: Foundation of a Course in Mathematics and Art

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In April of 2004, I had the pleasure of giving a workshop at the AAAS/NSF-sponsored *Invention and Impact* conference in Crystal City, Virginia. The workshop was titled "What I Wish I Had Known in Art School." The title is also the underlying theme of a course I teach in Mathematics and Art at Indiana University. I developed the material for the course with the idea of addressing vague uncertainties I had as an art student. In the process, I discovered that competence as an artist depends strongly on one's ability to look at art and nature and interpret the observations mathematically.

The ideas, activities, and materials from the course have spread to several institutions through a series of weeklong workshops for teachers called Viewpoints, which I teach with Annalisa Crannell of Franklin & Marshall College, Lancaster, Pennsylvania. The course and Viewpoints are sponsored by the NSF-funded Mathematics Throughout the Curriculum (MTC) project at Indiana University.

In this chapter, I will give some background on the MTC project, describe the Mathematics and Art course and the Viewpoints workshops, and properly conclude by saying what it is I wish I had known in art school.

The MTC Project and Its Prototype Course

The Indiana University MTC project was initiated and sustained by a grant from NSF's Division of Undergraduate Education (NSF-DUE 9555408), through an NSF-DUE initiative called Mathematical Sciences and Their Applications Throughout the Curriculum (MATC). The MATC initiative began in 1994. Its stated purpose was to promote systemic improvements in undergraduate education by increasing student understanding of, and ability to use, the mathematical sciences. The NSF-funded projects were to require collaboration of faculty in the mathematical sciences with faculty in other disciplines and full support of the participating academic units. The projects were expected to result in national models of undergraduate curricula and instructional approaches that would have a profound impact on the participating and adopting institutions.

In 1996, NSF accepted the Indiana University MTC proposal. Fellow MATC awardees included the University of Pennsylvania, Rensselaer Polytechnic Institute, Dartmouth College, a joint University of Nebraska/Oklahoma consortium, the State University of New York, and the United States Military Academy.

Besides the eight campuses of the Indiana University system, MTC participants include faculty from Franklin & Marshall College and the University at Buffalo. MTC currently continues on a no-cost extension.

The primary aim of MTC is truly interdisciplinary course development to correct undergraduate students' perception that mathematics is something isolated from, if not irrelevant to, other disciplines, the real world, and their own ambitions and goals. To this end, MTC required its course developers to

- motivate the learning of mathematics by linking it to goals of interest to the students
- require interdisciplinary work in groups of students with diverse skills
- require students to learn presentation skills for future use and for the sense of satisfaction and closure known well to the researcher but often lost in traditional course
- disseminate these approaches nationwide through workshops, presentations at national meetings, scholarly publications, and the development of course materials.

The prototype course for the MTC project is a course called Analytical Problem-Solving (APS), developed and taught by mathematician and project principal investigator Daniel Maki and business professor Wayne Winston at Indiana University's Bloomington campus. APS had already become a success by the time of the grant proposal and was seen as a proof-of-concept course by NSF. The course has continued to flourish since its inception.

The majority of APS students are liberal arts majors in the Liberal Arts Management program who wish to learn how mathematics is used in business. The course is based on the idea that most students will appreciate mathematics more if they see an immediate use for it. With this in mind, the entire course is built around group projects. In addition, it has the following distinctive features that students find especially appealing: Real problems, obtained from businesses and governmental units, are used to motivate the mathematical topics covered. The course is project-driven with topics matching the knowledge needed to complete the projects.

All mathematical topics taught in the course are used immediately in the projects; thus, the students see the usefulness of mathematics in solving a diverse set of real-life problems.

Students work in teams using computer software, primarily Microsoft Excel, to implement the mathematical tools developed in the class for the projects.

Students are required to document, explain, and defend their mathematical work, thereby honing their communications skills and providing them with a sense of closure and accomplishment.

A typical APS project was the scheduling of tellers at a local credit union. Students began by collecting data on service times. Then they used hourly and daily customer data to develop a model that could be used to forecast the workload at the credit union at any time. Next, queuing theory was used to determine, as a function of the forecasted workload and service rate, the number of tellers needed at any given time. A user-friendly spreadsheet was developed that could be used by the credit union personnel manager to determine manpower needs. Finally, the students developed a linear programming model to determine the minimum cost of scheduling tellers to meet forecasted manpower needs. This single project required the students to use basic statistics, regression analysis, queuing theory, advanced spreadsheet techniques, and linear programming.

Thus, from the students' point of view, the main goal of the project was not to pass an exam, but to produce a spreadsheet that could be (and actually has been) used by the credit union to minimize its costs and better serve its customers. In other words, the students acted as real consultants conducting original research. Similar APS consulting projects have been done for the Indiana State Department of Health, K and W Corporation, A. C. Nielson, Otis Elevator, Summit Health Care, Cummins Engine, Steak n Shake, Applied Composites Engineering, Bedford Machine & Tool, and Applied Laboratories (pharmaceutical and health care products). More than one APS student who has gone on to a successful career in business has reported that the most impressive part of their first job interview was the discussion of their APS consulting project.

Mathematics and Art: Real Art and Real Math

The success of the APS course became the yardstick for the two dozen subsequent MTC courses, which linked mathematics with subjects as diverse as biology, linguistics, criminal justice, physics, business, economics, history, journalism, chemistry, biomechanics, and the speech and hearing sciences. (Articles about these courses can be found in the online MTC project newsletter at www.indiana.edu/~iubmtc/newsletter.) In particular, the bar was set for the Mathematics and Art course: the applications would have to be of authentic interest to artists and contain challenging mathematics at an appropriate level.

Working in collaboration with fellow mathematician Annalisa Crannell, I drew upon my former 13-year career in fine arts to serve as the "art half" of our course development team. The core of my contribution is the text "Lessons in Mathematics and Art," which is currently available on the Viewpoints website at php.indiana.edu/~mathart/viewpoints. Crannell and I have used the text for several years now in mathematics courses for liberal arts majors at our respective institutions. In keeping with the philosophy of the APS course, "Lessons" concentrates on applications of mathematics that are of authentic value to artists and not just of scholarly interest to mathematicians.

There are math-for-liberal-arts texts on the market that link art-related activities and mathematics: mainly symmetry, tilings, and Platonic solids. There have also been a growing number of art-related math courses that focus on these topics. "Lessons," on the other hand, takes a different approach that makes it unique.

Unlike survey texts, "Lessons" deals with just two topics: perspective geometry (related to drawing manmade forms) and fractal geometry (related to drawing natural forms). The problems in "Lessons" are real art problems, and that is why art educators as well as math educators have embraced its approach. Art schools do not teach symmetry groups, tilings, or Platonic solids. On the other hand, all art majors (and many other students) take drawing courses and learn some basic perspective. "Lessons" builds on knowledge and interest the students already have and leads them to grapple willingly with the mathematics behind it.

The problems in "Lessons" are also real math problems. As the participants in the *Invention and Impact* workshop discovered, these problems can challenge even professional mathematicians, yet the level of the mathematics is entirely appropriate for the intended audience of liberal arts majors. As the workshop illustrated, a well-chosen perspective problem has the following properties:

- The problem is natural and easily understood.
- The problem has multiple solutions of varying difficulty and applicability.
- The solutions admit multiple proofs, both geometric and algebraic.
- Once arrived at, the solutions are easy to remember and rewarding to use.
- The solutions capture the essence of mathematical research and discovery.

In an authentic and uncontrived way, problems in perspective embody each of these characteristics and do so at a level that is accessible to every student. The same can be said of well-chosen applications of fractal geometry. (For a discussion of the use of fractal geometry, see Crannell and Frantz [2000].)

For the sake of brevity, let us stay with the topic of perspective and see how the course emulates another aspect of the APS course: activity-based learning. A major deficiency in the traditional method of teaching perspective, as corroborated by artists at the Viewpoints workshops, is that no one ends up understanding what perspective really is, because no one ever "does" perspective, by which we mean the following. The first day of a "Lessons" class, or the first morning of a Viewpoints workshop, begins with groups of students applying masking tape to windows to make drawings of the outside world. Each group is led by an "art director" who, rooted to a particular spot and viewing with only one eye, directs them to trace the objects he or she sees out the window. Days before, the instructor has carefully chosen the windows, prioritized the key objects outside them for drawing, and picked the viewing spots for the art directors. Thus, the students first encounter projective functions and perspective by "being" projective functions and "doing" perspective. The mathematical formulation comes later in a natural and meaningful way.

Thanks to the careful orchestration of the instructor, some basic principles of perspective begin to appear on the windows. There are image lines that converge to vanishing points although their pre-images are parallel; other image lines are parallel like their pre-images; and the important geometric relationship between the art director and the drawing becomes clear. Every subsequent viewer of the masking tape drawing must close one eye and locate the exact position of the director's eye (the correct viewpoint) or risk seeing a distortion. Ignorance of viewpoint is the reason for the apparent "over-perspectivized" distortion in beginners' perspective drawings. Beginners almost invariably choose (without knowing it) a viewpoint absurdly close to the paper, and thus everyone looks at the drawing from the wrong place, including the artist.

In later classes, the window-taping phenomena are formulated and verified mathematically, both with and without coordinates. When students gain some experience with the geometry of perspective, they are ready for another important activity: perspective viewing in an art gallery. This activity is roughly the inverse of the window-taping problem: given a work in perspective, how can one find the correct viewpoint, and what does the art look like from there?

These questions come from the science of photogrammetry, and the answers are surprising. Using geometry and facts known about the objects in the paintings (buildings have right angles, tiles are squares, etc.), the viewpoints can usually be located using only a pair of shish kebab skewers or lollipop sticks from a craft store. Although not as close as beginning art students inadvertently put them, the correct viewpoints are often quite close to the paintings, so that essentially no one ever views the paintings from the proper location. When the correct viewpoint is assumed and the painting is viewed with one eye, a competent perspective work will often appear, amazingly, three-dimensional and subsequently seem flat and ordinary from everywhere else in the room. This phenomenon continually surprises and delights students and instructors alike, and it is a skill they can enjoy for the rest of their lives.

There is plenty of mathematics here, too. Perspective was the virtual reality peep show of the Renaissance. Sadly, we have forgotten how to peep, but this is job security for the math teacher (although there are still galleries in Italy with tiles on the floor that show viewers where to stand). There are good art posters that can be analyzed more precisely in the classroom to answer all sorts of interesting questions. If we know where the artist stood, how high was the building shown in the picture? If we can estimate the height of the building, from where did the artist paint? Was the artist on a balcony? How high was the balcony? Was an aerial photograph taken from low altitude, or from high altitude and magnified? The difficulty of the analysis can vary greatly with the image, giving the instructor a wide range of options.

Confidence in the activity-based approach to teaching students implies that the same approach should be successful when teaching teachers. This has proven to be the case in the Viewpoints mathematics and art workshops.

The Viewpoints Workshops

The first Viewpoints workshop was held in 2000 and additional workshops were held in 2001, 2003, and 2004. There are two components of the workshops: a five-day residential workshop in the summer at Franklin & Marshall College and a one-day follow-up workshop in the fall at Indiana University. The summer workshops are intensive, hands-on sessions that train undergraduate instructors in the ideas and activities described in "Lessons" and used in the Mathematics and Art courses taught by Annalisa Crannell and myself. Each participant is provided with a free copy of the text. At the fall workshops, the summer participants present the results of their use of Viewpoints ideas and activities in their classes.

"Lessons" has continued to evolve, based on our experience with our own Mathematics and Art classes and on feedback from Viewpoints participants who have used it in their classes. Because of its emphasis on techniques that artists can actually use in their work, Viewpoints graduates have used "Lessons" not only in undergraduate liberal arts mathematics courses, but also in mathematics courses for art majors.

Instructors have used all or part of the text as the basis for liberal arts math courses at more than 17 institutions, including Clarion University, DePauw University, the College of St. Benedict, Arapahoe Community College, Goshen College, Chadron State College, the University of Rio Grande, Sul Ross State University, Joliet Junior College, the College of Charleston, Christian Brothers University, Marshall University, Centenary College, Mohawk College (Canada), George Brown College (Canada), Wheaton College, and St. Joseph's University.

In addition, "Lessons" has been used in courses and course development for art majors at two universities: 1) Steven Gendler (mathematics) and Jim Rose (art) have used "Lessons" as a text for a mathematics course for art majors at Clarion University, and 2) Barbara Reynolds and Andrius Tamulis (mathematics) and Peter Galante and Teri Wagner (art) at Cardinal Stritch University in Milwaukee are planning a two-semester mathematics course for art majors that will use the text.

Thus, the Viewpoints workshops have been instrumental in disseminating the text. In 2001, the Mathematical Association of America's Professional Enhancement Program (PREP) honored Viewpoints by supporting it as one of their first professional development workshops. Evaluator Barbara Edwards of Oregon State University wrote in her evaluation summary for PREP, "[The data] reveal at least a 74% extended success rate, which is very good for a one-week workshop."

What I Wish I Had Known

What I missed out on in art school was not so much a specific list of mathematical facts, but rather the knowledge of how beautifully mathematics and art can complement each other. Fortunately, the many Viewpoints participants seem to feel the same way. The continuing success of Viewpoints has inspired Annalisa Crannell and myself to plan an expanded version of "Lessons," along with an accompanying instructor's manual that will act as a "workshop in a book." Our goal is to reach an ever-wider audience and share the excitement and rewards of teaching Mathematics and Art. Just recently, after finishing Viewpoints 2004, an enthusiastic participant (a mathematician) wrote us to say,

> *"It was amazing—I couldn't believe how much cool and useful stuff you guys were able to cram in to one week. It was particularly interesting to have so many artists around. At the beginning of the week, I found myself noticing how differently we think, and then towards the end of the week, I found I was often surprised at how similar our disciplines really are."*

That's the kind of feedback that keeps us going, and it is exactly what I wish I had known in art school.

BIBLIOGRAPHY

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