RUSSIAN MATHEMATICAL CLASSES

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A highly developed system of mathematical education has been formed in Russia. Perhaps the word "system" is inappropriate, because many of its parts are not quite interconnected and do not correspond to one another in style and contents in a way such that we would call the resulting structure a system. These parts have been created by dozens and hundreds of universities, mathematical schools, various regional groups, and, particularly, enthusiastic persons. The system has arisen spontaneously and has numerous contradictions and imperfections. However, a certain degree of commonality and consistency can also be seen. The main parts of the system are (1) mathematical schools and classes, (2) city mathematical circles, (3) summer mathematical schools, (4) extramural mathematical schools (all-union extramural mathematical schools (AEMS)., Malyi Mehmat schools,[1] and certain others), (5) magazines (*Kvant, Mathematics in School, 1st September*, and other editions to aid teachers and gifted students), and (6) numerous Olympiads (all-Russian with all its stages, International Cities Tournament, regional, local, university organized, etc.—hundreds of competitions in total). We can thus separate all the activities into two parts: educationally oriented (activities 1–5) and competitions (activity 6). Education is the kernel of the system, while the competitions fill the classes and circles; in addition, they are the decorative elements of the system, giving the studies the atmosphere of a big festivity. Henceforth, I will call the system "Russian mathematical classes," thus underlining the leading role played by studies, but not forgetting that such a name is one sided and doesn't reflect the rich variety of ways of working with students.

History

Professional mathematical education started in Russia in 1701, when Peter the Great decreed the opening of the School of Mathematical and Navigational Science in Moscow. Mathematics there was taught by the first professional Russian mathematics teacher, Leontii Magnicki, author of the first Russian arithmetic book. We can fairly describe two hundred years of history of mathematical education in Russia up to the beginning of the twentieth century as exhibiting slow, but permanent, growth. Notable during the period were the foundation of the St. Petersburg Academy of Science; Leonard Euler's (and other big European scientists') work at the Academy; the appearance of the first Russian scientist, Mikhail Lomonosov; the inauguration of the first universities; and the work of Nikolai Lobachevskii, rector of Kazan University. During the reign of Elizabeth, daughter of Peter the Great and Russian empress from 1741 to 1761, the Moscow and St-Petersburg gymnasiums were joined by that of Kazan. The well-known eighteenth-century Russian poet Gavrila Derzhavin, one of the first to study at the Kazan gymnasium, writes in his biography that his mathematical tutor never asked for proofs of theorems, just the statements, because he never understood the proofs himself. This gives an idea of the level at which the gymnasiums began. Later, Alexander I issued a decree opening the gymnasiums in all the province's centers. An important detail of that period was that neither gymnasiums nor universities were popular among the people, so, to attract more people into education, various measures were taken, such as bestowing a personal title of nobility on, and offering an award to, every successful applicant upon entering the university. Later, in the 1860s, it was widely acknowledged that education had to be expanded to a greater population, and a flow of the nonnobility to the gymnasiums and universities took place.

After a time, certain ministers became concerned that soon there would not be enough people to plow the fields, and the attractive measures gave way to restrictive ones, but it was too late: The educational urge that had been planted in the society was impossible to stop. At the beginning of the twentieth century, the level of gymnasiums and "real colleges"[2] was such that, given the parents' welfare permitted, a young man from the country could, after graduating from one, become quite an educated person, well prepared for life in any social milieu of the time. Teachers and tutors from the educational system of the period did much to fight illiteracy in Russia. The mathematical education level was high enough. It is possible to judge it by the fact that the school book of the time can still serve as an example for us, while the mathematics teachers who had been formed at that time prepared the cadres who brought about the industrialization of the country. Today we can see that the

number of hours given for mathematics was actually not enough, at the time, that lead to a necessity for numerous pupils to hire private tutors (a situation that gave poorer students a chance to earn a small income). At the same time, in the beginning of the twentieth, the system of extracurricular mathematical education began to develop in Russia. A mathematical circle sponsored by Moscow University was free of charge and open to everyone. The magazine *Mathematical Education*, which illustrated the circle's work, began publication. The journal was edited by professor B. K. Mlodzeevskiy (whose son, A. B. Mlodzeevskiy, has become an extraordinary lecturer at the physics faculty).

The next period is the Soviet Era. The first part of it—before 1960— is the normal mass school development, where mass is the keyword. Its level surely fell, though was still supported by older teachers and traditions. There were several centers of mathematical culture: Moscow, St. Petersburg (although the opposite order would probably be fairer), and certain regional centers in the smaller areas' capitals. In 1930, new forms of extramural work with students appeared: circles and contests, attached to the universities and, later, at the towns' education departments. The second part of the Soviet period, from 1960 to 1980, was characterized by an expansion of high mathematical culture over the entire country, not just in the capitals. In the 1960s, the mathematical magazine *Kvant* appeared, the contests became all-Union Olympiad, and mathematical schools and classes arose, as did AEMS (All-Union Extra-mural Mathematical School). Systems like that of mathematical education were developing in parallel branches of science—physics, chemistry, and biology—but they have never reached the level attained in mathematics. So, what can explain that explosive development of the 1960s? For those who worked in mathematical classes and circles at that time (including myself), it was clear that the people who strove to get into those classes and circles differed radically from those who entered in previous years. A. S. Kronrod, one of the professors who established the mathematical schools and a battlefront veteran, once said, "These are the children of the Victory. Even their birth is a result of the Victory. And they've brought to school this feeling of the Victory." This great increase in interest and achievement is surprising, but the small decrease that followed is not.

Since 1980 the urge for education has diminished. The education system survives—with difficulty—through the internal energy of people. Although the level of education has fallen somewhat, it is still rather high compared with the level of education in other countries and regions in the modern world, even developed countries. In recent decades, the unique system of working with advanced students remained in place, but at the same time, the level of mass mathematical education has fallen gradually. The reason, it seems to me, is that for many years no strong students would enter pedagogical universities and colleges, perhaps because of the extremely low salaries of teachers. Not only have teachers' standards of living suffered, but the whole educational system is under threat, including the governing bodies. Reasonable, honest workers are seen less and less in the teaching profession. Their heroic fight for the preservation of the educational system contrasts more and more with the apathy of the common bureaucracy that long ago became a bin for useless and unfit workers who act out their own ideas in accordance with bureaucratic norms. The only hope is that it's impossible to govern our country and that somehow the reasonable forces will prevail.

How do mathematical schools and classes appear today?

The number of mathematical schools (or mathematical classes in schools that are not wholly mathematical) is to be counted in the hundreds. There is no unified system with common requests and approaches. If, in a certain settlement, there is an enthusiast who is ready to spend his or her time organizing a mathematical school (or classes), such a school (or such classes) will likely appear in that settlement. The level of the school would depend on local conditions and would also be seriously affected by the range of scientific interests of the organizing person. Our educational legislation allows every school several hours a week to use as it pleases in order to enforce education in a certain area. The organizers of mathematical classes use this opportunity. In the majority of cases, the level of such classes is not extremely high; however, it is still higher than in other schools, for two reasons: First, the teachers of those classes are usually more energetic than teachers of other classes, and second, more gifted students usually try to study for those classes affording the teachers a chance to demand more of their students. Scientific aims of such teachers are (mostly) concluded in preparation for university entry exams and contests. There are, however, extra-class mathematical schools (no more than a dozen, in my opinion, though there is no obvious boundary. These are №239 and №533 St.Petersburg schools, №2, №25, №57, №179, №1543 Moscow schools, Specialized Educational Scientific Center (SESC) at the Moscow State University and certain others. The principal difference between the two types of schools is that young and creative mathematicians take an active part in the educational process of the former. Some of them are working in modern, developing areas of mathematics; others are finding something new in more classical branches, such as planimetry. These people are all very different, and that means that the face of the schools they are patronizing differs as well. Certain schools are targeting the International Mathematical Olympiad,

others proclaim their target to be preparation for entry exams, and still others are oriented to prepare future scientific workers in mathematics or adjacent branches. However, all these targets exist at once in the strongest schools.

Certain functioning principles of mathematical schools and classes

The discussion that follows applies mostly to the kinds of schools discussed in the previous section. It will not do much harm if the general principles I set forth reflect only the views of my fellow thinkers and me — it is better to formulate them than to keep silent. The principal characteristics of mathematics classes are thoroughness, deliberation, and independence. Several key subjects are included in the program, which does not, of course, cover all of mathematics. Besides the standard school subjects, included in the curriculum are the elements of calculus, the theory of algorithms, and certain parts of higher algebra. Usually, the calculus is what goes on best of all, as it can invoke interest in the majority of pupils for a long time. However, the selection of subjects depends mostly on the teachers and their ability to be deeply interested in the subject and the students' work on it. Thoroughness implies that a subject is not covered in a temporary mode (like "anyway, you will learn it better during your higher education"), but in a final one (though this does not preclude returning to it at a new level later). When a subject is not covered thoroughly, it leads to loss of interest. A student who has missed a point here and a point there will end littering his or her studies to an extent where he will find it disgusting to live in it any more. By contrast, thoroughness helps the student find new interests in usual things again and again. The major role of a teacher is not to tell and explain, but to check everything thoroughly and examine all mistakes, keeping all the time a sincere interest in all the student's successes. It is this interest that is the major stimulating arms in the possession of a teacher, and not the marks; they can stimulate something, but usually not what is necessary.

Deliberation means that every difficulty receives as much time as is needed to overcome it. The trouble is not when little material is covered, but when there is a deadline by which something must be "covered," no matter how well or badly. This is a trouble, as it means that, in fact, nothing at all is covered and the result will be the loss of interest on the part of both students and teachers. Independence means that a significant part of the theoretical material, and sometimes almost all of it, is covered by the students themselves: On their own, they prove (or refute) the majority of theorems and problems that are presented to them. When a teacher starts lecturing, it is inefficient: Beginners usually do not understand any mathematical language. For instance, few of the talented beginners—can see the difference between "For any C, there is an x larger than C" and "There is an x larger than any C." The reader can now imagine, how much will the pupils understand in a competent discourse of a qualified mathematician. Thus, the most important teaching instrument becomes the structuring of the material. A selective control during tests and exams is not needed, but a permanent and thorough structuring of the material. Toward this end, many teachers are needed. The solution is the involvement of students. At strong schools, there are usually several mathematics teachers working with a class. Strong students are being called up, often winners of serious contests. The students pleasantly recount their own mathematical childhood with the class. Their relationship is like that of an elder and younger brother, while the professional teachers are like parents. The presence of students in a school significantly modifies the climate of the school, overcoming the age barrier.

An important feature of strong mathematical classes and schools is the participation in their work by professional mathematicians. Why such participation is important is difficult to explain, but almost any biography of a well-known scientist confirms the Latin proverb *Omnes cellula ex cellula*, ("Cells always emerge from cells"), indicating that the tutor of such a scientist was another well-known scientist. The scientific belief of the team head may be founded on his or her own current experience, on memories of his or her past experience, or on somebody else's experience. The first alternative is the most valuable one. Strong mathematical schools have made much progress in their teaching methods, compared with traditional schools, which make up the majority of our universities. Indeed, currently, there is no question of presenting the school with the university methods of work, but rather of the universities borrowing some of what can be found at schools. It is worth noting that, among the mathematics classes' teachers, there is a widespread conviction that there should be no intentional preparation of students for mathematical contests. A good performance at a contest, it is believed, should be a side effect of the mathematical level attained, not the result of express studies of particular known types of problems and methods of their solutions. This doesn't mean that instructive contest-like problems, containing interesting methods and ideas should not be studied, but they should not be studied for the sake of winning contests: Strong students are far too precious to the nation to waste their time and energy on such a useless thing as the prestige of a city or country. The work performed at mathematical circles and extramural schools is mostly going marked by same principles, as that in mathematics classes, although the former two have fewer possibilities than the latter. It is almost impossible to cover a long, coherent

matter at a circle, as its membership is inevitably unstable. At an extramural school, the membership is stable, but personal contact between teacher and student is lacking. There are even fewer possibilities at the summer mathematical school, because of its brief duration. However, for a large number of Russian students, participation at the summer mathematical school turns out to be their only option for obtaining additional knowledge.

Some details about Olympiads

Mathematicians from Moscow and Leningrad began organizing the Olympiads in the early 1930s. The traditions of the first Olympiad were mostly opposed to those of school. For example, although a test at school aims to check the students' possession of the covered methods for solution of a given class of problems, a contest offers students atypical tasks for which no method is known beforehand. If the school tends to give top marks for the so-called "perfect" work, the Olympiad awards were given out for the solution of few, but difficult, tasks, while the writing requirements were minimal, limited by the need for the jury to understand the solution. But the main distinction between a contest and school was that the contest offered interesting problems that pique the students' desire to think about them even after the contest was over and that a participant was happy to tell classmates about. At the 11th Moscow Mathematical Olympiad a 10th-form student, Kolya Korst only handed in a drawing, without solution. However, the jury saw in the drawing a yet unknown and, at the same time, the shortest solution of one of the tasks given. The jury decided to award the student a third prize. N. N. Korst later became a theoretical physicist. At the 25th Olympiad, Dima Kajdan, a 9th-form student, had solved only one problem and hadn't even had enough time to write his solution down. He was permitted to tell his solution to one of the jury members (A. M. Leontovitch). The jury made its decision only after an additional explanation given by Kajdan a week later. Kajdan turned out to be the only participant to solve the problem—it was the most difficult one—and has received the second prize. D. Kajdan is now a well-known mathematician. These examples show how the scientific aspect of the contest usually prevailed over the entertainment one. Another detail to be looked at with attention is that, on a school test, the grade is usually determined by the number of mistakes. By contrast, in a contest, the grade is usually two-valued: "solved" or "not solved." There is, in addition, a "plus–minus" grade, meaning a solution with a serious, though easy-to-fill, gap. But a problem that is thus evaluated is still considered solved. A very interesting, though not yet popular, tradition has formed at the Leningrad contests, where the pupils present their solutions orally. If a problem cannot be considered solved at the first attempt, a second one is given, and, later, a third. If, finally, the problem is solved, it is thus taken into account, independently of the number of attempts. The idea of two-values is carried to the limit here.

The 60th All-Union Mathematical Olympiad took mostly the traditions of the Moscow and Leningrad contests. Nowadays, the organizer of the All-Russian mathematical contest is the Ministry of Education and Science of the Russian Federation. The All-Russian contest is an enormous arrangement, composed of school, district, city, region, zone, and, finally, all-Russian stages. In numerous cities, the local scientific institutions and universities take an active part in organizing the city and region contests. The winners of the all-Russian contest take part in the International Mathematical Contest. Among the defects of the All-Russian contest system is the low level of several local contests—so low that there are not enough qualified organizers. First, there is a lack of qualified people who can check the students' work at the required level. Other defects pertain mostly to the nature of the contest itself. Prime among these is the scant time allowed for the solution of difficult problems. Beginners are given five hours for their test—far more than enough time to finish. The same five hours, however, is given for the solution of difficult creative tasks, too little time for the complexity of the tasks involved, whereupon the contest becomes also a speed competition, which is against the scientific spirit. Another defect of the whole system, including the International Contest, is that almost every one of the thousands of participants at all stages of the contest is a loser at a certain stage. This, too, is against the main aim of the contests: to attract into mathematics talented youth from all over the world of thousands, not just individuals.

In 1980, the International Tournament of Towns began. Initially, it appeared as a sort of addition to the All-Union Olympiad, but later it found its own ecological niche and has since acquired independent importance. In the way it is carried out, it is an ordinary contest. Every participant does written work in the town where he or she lives. The tasks and the deadlines, as well as the evaluation of the results, are the same for all the participants. This is a relatively cheap arrangement compared with the big Olympiads, which invite the participants and their tutors from far away. The competition has four stages, organized on a yearly basis: two in autumn (basic and difficult) and two of the same in spring. Entry to all four stages is free and open to all who apply. The evaluation of the results of a student in a stage is done on only the three best resolved problems. Thanks to this rule, the students are able to solve the problems without hurrying. As a matter of fact, this same

rule applied in the first Moscow Olympiads. The final tally of points for a student over the four stages was the maximum of what he or she had had gotten at each of the stages. This rule helps lessen the nervous tension that is often excessive at traditional contests. It also allows a student, as well as a whole town, a possibility to take part in just some of the four stages. All four stages compose the first level of the Tournament, which is suitable for both beginners (the basic stages) and more advanced students and professional contesters (the difficult stages, where the tasks are at the difficulty level of international contests). Most recently, about 120 towns and 25 countries participated in the Tournament. The total population of the towns is about 100 million people, and the number of participants is on the order of 10 thousand. Approximately a thousand diplomas are issued yearly on behalf of the Central Organizing Committee. More than half of the participating towns are Russian. Sixty tournament prizewinners, who have obtained the highest results, are invited to the Summer Conference of the Tournament of Towns. At the conference, they get several creative research problems for a week and there are no prizes or places awarded at all. There are certain exigencies common to all the contests. A contest should have a scientific side as well as a sporting one. The first aspect must be up to the mark, while the latter should not be overemphasized. To meet its aims, a contest must be open to all interested students (otherwise, where will the new faces come from?) and the tasks should be interesting and remarkable, such as to make the pupil wish to try and solve them after he or she has failed to do so during the contest. The evaluation of the student's work must be thorough and of good quality, and the students should be informed, in as detailed a manner as possible, how their personal work has been evaluated. Finally, the results of a contest must be used properly. That is to say, the contest itself, when well organized, is like a good school, helping to discover students who, on their merits, should be invited to specialized circles and classes. To conclude this section, I would just like to observe that it is easy to organize a bad contest: Offer low-level tasks, use unqualified judges, etc. Unfortunately, there are too many contests in Russia, and there are not enough people to organize them well. A shift towards sportiveness and a lack of attention toward the educational aspect appears to be occurring. The reason is clear: The organizing of education demands a lot of strength and endurance.

The interaction of the parts of the system

All the parts of the system of mathematical education of students support each other. Winners of mathematical contests enter mathematics classes, circles, and summer and extramural schools. Graduates of mathematics schools and classes enter better universities. University students participate with interest in the work with mathematical classes and circles, and are probably the main activists of mathematical contests. While teaching, the students also learn—for example, to be teachers. Unfortunately, only few of them do become professional teachers, but the reason is obvious: the unsatisfactory social position of the teaching profession. Following are a few examples of useful interactions. Every year, on the last Sunday of September, the Lomonosov Tournament, a rather easy multidisciplinary contest, is held in Moscow. No prizes are awarded for first, second, etc., place at this tournament, but if a student has, for example, shown good results in physics and biology, he or she gets a diploma, citing the fact that it has been issued for good results in physics and biology. The tournament is quite popular, and its aim is to involve students in serious studies. Thus, the first Saturday after the Tournament the Malyi Mehmat system of circles begins its work. The dates have been adjusted to take full use of the Tournament for the replenishment of the circles. All of the Tournament's participants, who took part in the linguistics contest are invited to participate in a Linguistics and Mathematics Contest that takes place shortly afterwards. Those who did work in physics are invited to the physics circles attached to the Moscow State University, and those who did astronomy to the astronomy circles, functioning at the House of Scientific Creativity of Youth. Thanks to the contests, to the Tournament of Towns, and to other summer schools, most of the strongest Russian students get acquainted with each other even before they graduate. Still more important is that the same is true about their probable future tutors. Thus, most of the students entering the mathematics department of the Higher School of Economics or the Independent Moscow University are quite familiar to the teachers, and new enrollees have long ago ceased to be regarded as a gift of fortune. Another form of interaction of the system's parts is the invitation of the leading universities' teachers to the summer schools. Such intercourse covers up to a hundred Russian towns, and the word "province" no longer has its older dark meaning. However, a hundred towns is far from all of the country, and it is now time to speak of what the system has not yet attained, although it would like to and probably still could, given certain conditions.

The system's imperfections

Multiple interconnections between local activists make it possible to establish a system for the creation and distribution of books on mathematics for students who need additional literature. There are those who could become authors, and the problem is how to involve them in this work, given that they are busy with other tasks at the same time. Numerous towns are in need of regional mathematical circles, and even though there are people who could cope with the task of establishing them, they, too, are otherwise occupied, and it is necessary

to change the structure of their obligations. Even the Tournament of Towns is, in reality, a school that is at the moment rather badly organized and thus inefficient. Then, apart from the organizational, there is a substantial problem: Qualified supervisors are required to verify the students' written works, and there is a lack of such individuals. Yet again, those who might manage the job are busy with other work. However, there is still some other recourse: It is possible to influence the priority scheme that the teachers, students, and potential supervisors implicitly follow. It is also noteworthy that the Tournament of Towns is a system that helps to connect the local activists and permits the organization of larger arrangements. Thus, it is possible to organize schools for foreign participants in Russia as well as abroad; indeed, some Latin American countries and several other places claim to have established such schools. Another factor is that it is necessary to organize the interaction of mathematical teachers at all levels, (school and university alike) so that they exchange the information easily and freely, hold meetings and conferences over the Internet, and more. Serious technical support is needed for this task, however. If carried out, though, such informational exchange could unite local activists and provide them with a voice and power; certainly, its accomplishment would help to elaborate a clearer view of the country as a whole regarding the problems of mathematical education. Toward that end, it is necessary to work more with the central educational governing bodies and with the government in general so as to influence the decisions that are being made. It is also important to work more closely with the mass media. Finally, to realize all these possibilities, not only intellectual, but also financial, resources are needed, as it is impossible to secure voluminous and regular work on pure enthusiasm.

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[1] Mehmat — Faculty of Mechanics and Mathematics of Moscow State University. Malyi (small)

Mehmat is a circle for school pupils that functions attached to the above-mentioned faculty.

[2] *Realschule*, a type of secondary school.